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On the stability of thermally radiative magnetofluidynamic channel flow – an amendment

J. B. HELLIWELL

School of Mathematics, University of Bradford, Bradford, Great Britain

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In the course of further developments to account for temperature dependence of the dissipative and absorption coefficients the earlier solution for the case of constant coefficients, Helliwell [1], has been found to be faulty. An error has been made in the mathematical analysis which however does not invalidate the main conclusion of that paper, viz. that for two dimensional disturbances in channels with black walls at the same temperature, radiative heat transfer has no effect upon the stability of the flow since the additional eigenvalues introduced are always associated with stable disturbances.

This note serves to correct the error referred to above and presents amended values for the eigenvalues given in Tables 1–4 of the previous paper. The references all relate to the numbered equations of the earlier paper and the same notation is employed here.

The error which consists of the omission of a factor $(K^2 + 3\omega^2)$ first appears in equation (26) which should read

$$3\omega\left\{\frac{d^2}{dy^2} - (K^2 + 3\omega^2)\right\}Q_y - 48\omega^2\theta_1^2\left(\theta_1\frac{d}{dy} + 3\theta_1'\right)\theta = 0. \tag{26}$$

The analysis relating to Squire's theorem is unaffected and the next point at which an amendment is required is at equations (28) and (29). These become

$$\phi = \{iK(v_1 - c) + 16\gamma\epsilon\omega\theta_1^3/\beta\}\theta, \tag{28}$$

$$\frac{d^2\phi}{dy^2} - \left\{\frac{i\beta K(v_1 - c)(K^2 + 3\omega^2) + 16\gamma\epsilon\omega K^2\theta_1^3}{i\beta K(v_1 - c) + 16\gamma\epsilon\omega\theta_1^3}\right\}\phi = 0. \tag{29}$$

The boundary condition (31) needs modification and should be

$$\left(\frac{2}{\varepsilon_\pm} - 1\right)\frac{d\phi}{dy} \pm \frac{3\omega}{2}\phi = 0, \quad \text{at } y = \pm 1, \tag{31}$$

all other forms being unaffected.

As a consequence of these changes it is no longer possible to derive a special result for the case when $K = 1$. Nevertheless a numerical solution may be obtained and, when black walls at the same temperature are considered, Section 6 of the earlier paper is modified so that the

TABLE 1
Imag (*c*). *Poiseuille flow*

	Odd eigenvalues						Even eigenvalues		
ω	1.0			0.1			0.1		
R	10^3	10^5	10^7	10^3	10^5	10^7	10^3	10^5	10^7
K									
0.2	−0.430	−0.137	−0.134	−0.149	−0.016	−0.014	−0.149	−0.016	−0.014
0.4	−0.215	−0.069	−0.067	−0.075	−0.008	−0.007	−0.075	−0.008	−0.007
0.6	−0.143	−0.046	−0.045	−0.050	−0.005	−0.005	−0.050	−0.005	−0.005
0.8	−0.107	−0.034	−0.033	−0.037	−0.004	−0.003	−0.037	−0.004	−0.003
1.0	−0.086	−0.027	−0.027	−0.030	−0.003	−0.003	−0.030	−0.003	−0.003
1.2	−0.072	−0.023	−0.022	−0.025	−0.003	−0.002	−0.025	−0.003	−0.002
1.4	−0.061	−0.020	−0.019	−0.021	−0.002	−0.002	−0.021	−0.002	−0.002
1.6	−0.054	−0.017	−0.017	−0.019	−0.002	−0.002	−0.019	−0.002	−0.002
1.8	−0.048	−0.015	−0.015	−0.017	−0.002	−0.002	−0.017	−0.002	−0.002

$\gamma = \frac{5}{3}$, $\epsilon = 1$, $\beta = 1000$, $M = 0$.

TABLE 2
Imag (*c*). *Odd eigenvalues. Hartmann flow*

J	−0.5			0			0.5			1.0		
R	10^3	10^5	10^7	10^3	10^5	10^7	10^3	10^5	10^7	10^3	10^5	10^7
K												
0.2	−0.829	−0.034	−0.014	−0.613	−0.028	−0.014	−0.486	−0.024	−0.014	−0.467	−0.024	−0.014
0.4	−0.414	−0.017	−0.007	−0.306	−0.014	−0.007	−0.243	−0.012	−0.007	−0.233	−0.012	−0.007
0.6	−0.276	−0.011	−0.005	−0.204	−0.009	−0.005	−0.162	−0.008	−0.004	−0.156	−0.008	−0.004
0.8	−0.207	−0.008	−0.003	−0.153	−0.007	−0.003	−0.121	−0.006	−0.003	−0.117	−0.006	−0.003
1.0	−0.166	−0.007	−0.003	−0.122	−0.006	−0.003	−0.097	−0.005	−0.002	−0.093	−0.005	−0.003
1.2	−0.138	−0.006	−0.002	−0.102	−0.005	−0.002	−0.081	−0.004	−0.002	−0.078	−0.004	−0.002
1.4	−0.118	−0.005	−0.002	−0.088	−0.004	−0.002	−0.069	−0.003	−0.002	−0.067	−0.003	−0.002
1.6	−0.104	−0.004	−0.002	−0.077	−0.003	−0.002	−0.061	−0.003	−0.002	−0.058	−0.003	−0.002
1.8	−0.092	−0.004	−0.002	−0.068	−0.003	−0.002	−0.054	−0.002	−0.001	−0.052	−0.003	−0.001

$\gamma = \frac{5}{3}$, $\epsilon = 1$, $\beta = 1000$, $\omega = 0.1$, $M = 5$.

TABLE 3
Imag (c). Odd eigenvalues. Hartmann flow.

β	100			10000		
K \ R	10^3	10^5	10^7	10^3	10^5	10^7
0.2	−0.912	−0.142	−0.130	−0.272	−0.009	−0.0015
0.4	−0.456	−0.073	−0.067	−0.136	−0.005	−0.0007
0.6	−0.304	−0.048	−0.045	−0.091	−0.003	−0.0005
0.8	−0.228	−0.036	−0.033	−0.068	−0.002	−0.0004
1.0	−0.182	−0.029	−0.027	−0.054	−0.002	−0.0003
1.2	−0.152	−0.024	−0.022	−0.045	−0.002	−0.0002
1.4	−0.130	−0.021	−0.019	−0.039	−0.001	−0.0002
1.6	−0.114	−0.018	−0.017	−0.034	−0.001	−0.0002
1.8	−0.101	−0.016	−0.015	−0.030	−0.001	−0.0002

$\gamma = \frac{5}{3}$, $\epsilon = 1$, $\omega = 0.1$, $J = 0.5$, $M = 5$.

TABLE 4
Imag (c). Odd eigenvalues. Hartmann flow.

ϵ	0.1			10		
K \ R	10^3	10^5	10^7	10^3	10^5	10^7
0.2	−0.049	−0.0024	−0.0014	−4.859	−0.241	−0.135
0.4	−0.024	−0.0012	−0.0007	−2.430	−0.120	−0.067
0.6	−0.016	−0.0008	−0.0004	−1.620	−0.080	−0.045
0.8	−0.012	−0.0006	−0.0003	−1.215	−0.060	−0.034
1.0	−0.010	−0.0005	−0.0003	−0.972	−0.048	−0.027
1.2	−0.008	−0.0004	−0.0002	−0.810	−0.040	−0.022
1.4	−0.007	−0.0003	−0.0002	−0.694	−0.034	−0.019
1.6	−0.006	−0.0003	−0.0002	−0.607	−0.030	−0.017
1.8	−0.005	−0.0003	−0.0002	−0.540	−0.027	−0.015

$\gamma = \frac{5}{3}$, $\beta = 1000$, $\omega = 0.1$, $J = 0.5$, $M = 5$.

boundary condition (32) becomes

$$\frac{d\phi}{dy} + \frac{3\omega}{2}\phi = 0, \quad \text{at } y = 1. \tag{32}$$

Furthermore

$$\begin{aligned} f &= C\{[Kb\beta + 16\gamma\epsilon\omega\theta_1^3][Kb\beta(K^2 + 3\omega^2) + 16\gamma\epsilon\omega K^2\theta_1^3] \\ &\quad + K^2\beta^2(K^2 + 3\omega^2)(v_1 - a)^2\}, \\ g &= 48C\gamma\epsilon\omega^3 K\beta(v_1 - a)\theta_1^3, \\ C &= 1/\{[Kb\beta + 16\gamma\epsilon\omega\theta_1^3]^2 + K^2\beta^2(v_1 - a)^2\}. \end{aligned}$$

Also

$$E = 1, \quad F = 0, \quad G = 3\omega/2, \quad H = 0.$$

The same iterative scheme may be employed as before. The calculation is started by taking as first approximation to the eigenvalue when $K = 1$ (in cases when $\omega = 0.1$) that computed for the previous paper, since then the omitted factor $(K^2 + 3\omega^2)$ has the value 1.03 which is nearly unity. Tables 1–4 give the results of the calculations.

REFERENCE

[1] J. B. Helliwell, On the stability of thermally radiative magnetofluidynamic channel flow. *J. Eng. Maths.* 11 (1977) 67–80.